Two Dimensional Temperature Distribution Resulting From Propagation of Light Beam Through Amplifying Medium

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The Green function theory was used to determine the temporal behavior of the two dimensional temperature distribution in a cylindrical rod placed in the focal axis of a cylinder of elliptical cross-section where in the other axis a flash lamp was mounted to pump the rod. The rod which was subjected in axial direction to a laser radiation to be amplified was assumed to be heated from the volume absorption of both the pump radiation and the laser radiation. Mathematical expression for the temperature distribution was obtained considering linear behavior of the rod, different focusing configuration of the pump beam and different radii of the laser radiation. As an illustrative example computation was carried out on a Ruby rod.

1. Introduction:

Since the development of lasers, a lot of applications based on their particular properties have been made possible in the field of science and engineering. High power lasers which are applicable in plasma generation, change of the phase of the absorbing material [1] and producing pn-junction [2] are already gaining acceptance in material processing areas such as spot welding, cutting and drilling of holes [3-5]. This comes up from the theoretically reliable extreme small-focused spots that allow material processing to be localized at well defined locations in the target. Thus, the study of thermal effects of the laser on the solid target when the light is absorbed is necessary because it gives the required information to control material processing or to avoid damage of the irradiated surfaces as in case of laser mirrors [6 & 7]. The deleterious effects of the pump power induced heat on the performance of solid state lasers were studied [8-14].

As high output power is required, just for instance for laser machining processes, and the output power of the laser oscillator is not great enough the radiation will be guided through an amplifier that has to be pumped. The radiation to be amplified and the pump radiation both heat the amplifier and this may lead to phase change, thermal stresses leading to cracks and variation of the refractive index and under circumstances to vary the plane form of the input and output surfaces of the amplifier. These effects may lead to damage the amplifier or varying the front surface of the radiation leading to inaccurate laser machining.

To avoid these changes a pre-study of the spatial and temporal temperature distribution has to be carried out in linear and nonlinear medium. The results of this study will be later applied in equations concerning the thermal stresses, the results of which will be necessary to determine the variation of the refractive index. With the aid of Maxwell equations the wave propagation in such an optically deformed amplifier will be determined. From the obtained results it will be hoped to find a way to correct the front surface of the wave and so to increases the accuracy of laser machining.

2. Theory:

Assuming that a homogeneous isotropic cylindrical shaped target with circular cross-section was illuminated in axial direction with a laser beam that has to be amplified. Due to this assumption the target was pumped radially with a radiation originating from a gas discharge tube located in the focal axis of a cylinder with elliptical cross-section having reflecting surface. The target was placed in the other focal axis where the pump radiation was differently focused. The laser beam, which has a maximum temporally coinciding with that of the pump pulse, was considered to have the same temporal profile but with different pulse duration. Its spatial distribution was considered to have either a Gaussian shape of different constant or increasing cross-section along the propagation direction (Gaussian beam). Moreover it was assumed that a part of the pump radiation and that of the amplified laser were absorbed during their propagation process within the target which was cooled radially at the outer cylinder surface.

Considering no plasma formation at the irradiated surface, and negligible multiphoton absorption, the equations governing the temperature distribution are given by:

1) The heat diffusion equations in cylindrical coordinates

i- Originating from the pump beam

$$\frac{\partial^2 T_p(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T_p(r,t)}{\partial r} + \frac{1}{k} g_p(r,t) = \frac{1}{\alpha}\frac{\partial T_p(r,t)}{\partial t} \qquad (1)$$

ii- Originating from the laser radiation

$$\frac{\partial^2 T_L(r,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T_L(r,z,t)}{\partial r} + \frac{\partial^2 T_L(r,z,t)}{\partial z^2} + \frac{1}{k} g_L(r,z,t) = \frac{1}{\alpha}\frac{\partial T_L(r,z,t)}{\partial t} \qquad (2)$$

2) The boundary condition describing the cooling at the outer radius of the rod.

$$-k\,\frac{\partial T_t(r,z,t)}{\partial r}\Big|_{r=R} = h\,T_t(r,z,t)\Big|_{r=R} \qquad (3)$$

with $T_t(r,z,t) = T_L(r,z,t) + T_P(r,t)$, (4)

where g_p (r,t) is the rate of energy generated from the absorbed part of the pump beam per unit volume, $g_L(r,z,t)$ is the rate of energy generated from the absorbed part of the amplified laser radiation per unit volume, k is the thermal conductivity of the material, $T_L(r,z,t)$ is the temperature distribution generated form the absorbed part of the amplified laser beam, T_P (r,t) is the temperature distribution generated from the absorbed pump radiation, $\alpha = k/(\rho\, C_p)$ is the thermal diffusivity of the material; ρ is the mass density of the material, C_P is the specific heat of the material at constant pressure and h is the heat transfer coefficient.

To determine the temperature distribution within the target resulting from the pumping process, the Green's function approach for solving non-homogonous transient heat conduction was applied. This gives according to [15],

$$T(r,t) = \int_R r' G(\vec{r},t\,|\vec{r}',\tau)\big|_{\tau=0} F(\vec{r}')dv' + \frac{\alpha}{k}\int_{\tau=0}^{t} d\tau \int_R \vec{r}' G(\vec{r},t\,|\vec{r}',\tau) g(\vec{r}',\tau)dv'$$

$$+\,\alpha \int_{\tau=0}^{t} d\tau \sum_{i=1}^{N} \int_{S_i} \vec{r}' G(\vec{r},t\,|\vec{r}',\tau)\big|_{r'=r_i} \frac{1}{k_i} f_i(\vec{r}',\tau)dS_i' \qquad (5)$$

where

$G(\vec{r},t\,|\vec{r}',\tau)$ is the Green function, R is the entire volume of the considered region;

S_i is the boundary surface of the region R, i=1,2,….., N; N is the number of continuous boundary surfaces;

dv' and ds_i' are the differential volume and surface element in the $\vec{r}'$ variable respectively, $F(r')$ is the initial temperature distribution, $g(\vec{r},t)$ is the rate of energy density responsible for the heat generation, $f_i(\vec{r}',\tau)$ is the boundary conditions function.

To get the Green's function one has to solve the homogenous transient heat conduction problem obtained from the following inhomogeneous set of equations, after setting $g(\vec{r},t), f_i(\vec{r}',\tau)$ equal to zero and substituting $\Psi(\vec{r},t)$ for $T(\vec{r},t)$

$$\nabla^2 T(\vec{r},t) + \frac{1}{k} g(\vec{r},t) = \frac{1}{\alpha}\frac{\partial T(\vec{r},t)}{\partial t} \quad \text{in region R, } t>0 \qquad (6)$$

$$k_i \frac{\partial T(\vec{r},t)}{\partial n_i} + h_i T(\vec{r},t) = f_i(\vec{r},t) \qquad \text{on } S_i, \qquad t > 0 \tag{7}$$

$$T(\vec{r},t) = F(\vec{r}) \qquad \text{for } t = 0 \quad \text{in R} \tag{8}$$

This gives after some mathematical manipulations [15] :

$$\Psi(\vec{r},t) = \int_R K(\vec{r},\vec{r}',t).F(\vec{r}')dv' \tag{9}$$

$K(\vec{r},\vec{r}',t)$ is the kernel of integration. The Green's function is obtained at τ = 0 by setting [15]

$$G(r,t\,|r',\tau)\big|_{\tau=0} = K(r,r',t) \tag{10}$$

$G(\vec{r},t\,|\vec{r}',\tau)$ for the transient heat conduction is obtainable from $G(\vec{r},t\,|\vec{r}',0)$ by replacing t by (t-τ) in the latter [15].

Applying this procedure on Eqn. (1) subjected to the boundary condition, one gets:

$$\Psi(r,t) = \sum_{m=1}^{\infty} e^{-\alpha\beta_m^2 t} \frac{1}{N(\beta_m)} J_o(\beta_m r) \int_o^R r' J_o(\beta_m r') F(r') dr' \tag{11}$$

with β_m are the positive roots of the equation $\beta_m J_o'(\beta_m R) + HJ_o(\beta_m R) = 0$ and

$$N(\beta_m) = \int_0^R r J_o^2(\beta_m r) dr = \left[\frac{2\beta_m^2}{J_o^2(\beta_m R) R^2 (H^2 + \beta_m^2)}\right]^{-1}$$

where H=h/k, $J_o(\beta_m r)$ and $J_o'(\beta_m R)$ are the Bessel functions of zero order and first order, respectively. $G(r,t\,|r',\tau)\big|_{\tau=0}$ is given by

$$G_p(r,t\,|\,r',\tau)\,|_{\tau=0} = \sum_{m=1}^{\infty} e^{-\alpha\beta_m^2 t} \frac{1}{N(\beta_m)} J_o(\beta_m r) J_o(\beta_m r') \tag{12}$$

Applying this procedure on the present case ($F(\vec{r}) = 0$ and $f_i(\vec{r}',\tau) = 0$) one gets:

$$T_p(r,t) = \frac{\alpha}{k} \int_{\tau=0}^{t} d\tau \int_R r' G_p(r,t\,|r',\tau)\, g_p(r',\tau)\; dr' \tag{13}$$

The temperature distribution resulting from the absorbed part of the amplified laser radiation described by Eqn. (2) and subjected to the boundary conditions,

$$-\frac{\partial T_L(r,z,t)}{\partial r}\Big|_{r=R} = H T_L(r,z,t) \qquad \frac{\partial T_L(r,z,t)}{\partial z}\Big|_{\substack{z=0\\ z=L}} = 0 \tag{14}$$

is found to be given by:

$$G_L(z,t\,|z',\tau)|_{\tau=0}=\frac{1}{2\sqrt{\alpha}\sqrt{\pi}\sqrt{t}}\left[\exp-\left[\left(\frac{2L-z-z'}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]+\exp-\left[\left(\frac{z+z'}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]+\right.$$

$$\exp-\left[\left(\frac{2L-z'+z}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]+\exp-\left[\left(\frac{z-z'}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]+\exp-\left[\left(\frac{2L+z-z'}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]+$$

$$\left.\exp-\left[\left(\frac{4L-z-z'}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]+\exp-\left[\left(\frac{z+z'+2L}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]+\exp-\left[\left(\frac{4L-z'+z}{2\sqrt{\alpha}}\right)^2\frac{1}{t}\right]\right] \quad (15)$$

Replacing t by (t-τ) in Eqn. (15) one gets $G_L(z,t\,|z',\tau)$ and, thus, $T_L(r,z,t)$ can be written as:

$$T_L(r,z,t)=\frac{\alpha.2\pi}{k}\int_{r=0}^{R}\int_{\tau=0}^{t}r'\sum_{m=0}^{\infty}\frac{2\beta_m^2}{R^2(H^2+\beta_m^2)}\frac{1}{J_o^2(\beta_m R)}\exp\left[-\alpha\beta_m^2(t-\tau)\right]$$

$$J_o(\beta_m r')J_o(\beta_m r)\int_o^L G_L(z,t\,|z',\tau).g_L(r',z',\tau)dr'dz'd\tau \quad (16)$$

- Determination of the heat generation function of the pump beam $g_p(r,t)$**:**

Setting the pump source and the rod to be pumped in the two focal axis of a cylinder with elliptical cross section and highly polished reflecting surface, leads to confining the radial symmetric irradiation into an area tending to zero, and therefore to an intensity tending to infinity.

Due to the diffraction of the radiation on the reflecting surface and the imperfection of the elliptical cross section, circular spot of radius R_o with its center located on the focal axis of the cylinder will be considered. Due to this assumption the differential equation of the intensity distribution in radial direction within the rod will be given by:

$$\frac{dI_p(\upsilon,r)}{dr}=-\frac{I_p(\upsilon,r)}{R_o+r}+\alpha(\upsilon)I_p(\upsilon,r) \quad (17)$$

The first term in the R.H.S. of Eqn. (17), which is responsible for the focusing process, is negative because the intensity decreases by increasing r value. The second term on the R.H.S., which is responsible for the attenuation, is positive due to the increase of the intensity with increasing r value. With $I_{p\upsilon}(\upsilon,R)=I_{op\upsilon}(\upsilon)$ the solution of Eqn. (17) can be written as:

$$I_{p\upsilon}(\upsilon,r)=I_{op\upsilon}(\upsilon)\tilde{g}_p(r,\upsilon)$$

with $\tilde{g}_p(r,\upsilon)=\dfrac{R_o+R}{R_o+r}e^{\alpha(\upsilon)(r-R)}$

$I_{p\upsilon}(\upsilon, r)$ is the spectral intensity distribution of the pump beam as a function of location r and frequency υ

$\alpha(\upsilon)$ is the attenuation coefficient as a function of frequency υ

r is the radius at which the intensity has to be calculated measured from the center of the rod,

R is the radius of the cylindrical rod to be pumped.

Because the factor $\frac{R_o + R}{R_o + r}$ is responsible for the focusing effect and not for the heating process the radial derivative of the exponential factor has to be considered alone for the heat generation, this gives

$$\frac{dI_{p\upsilon}(\upsilon, r)}{dr} = I_{op\upsilon}(\upsilon)\frac{R_o + R}{R_o + r}\alpha(\upsilon)e^{\alpha(\upsilon)(r-R)} \tag{18}$$

Considering the time dependence of the pulse shape to be given by

$$G_p(t) = \frac{(n+1)^{(n+1)}}{n^n}\frac{t}{\Delta t_p}(1 - \frac{t}{\Delta t_p})^n \quad n = 3 \quad 0 \le t \le \Delta t_p \quad [16] \tag{19}$$

the absorbed power density of the pump beam leading to heat the rod in case of highly polished surface is given by

$$g_p(r,t) = X\int_{\upsilon_1}^{\upsilon_2}\alpha(\upsilon)G_p(t)I_{op\upsilon}(\upsilon)\frac{R_o + R}{R_o + r}e^{\alpha(\upsilon)(r-R)}d\upsilon \tag{20}$$

where Δt_p is the duration of the pump beam, X is the part of the absorbed radiation which heats the rod, $I_{op\upsilon}(\upsilon)$ is the spectral intensity distribution given by Planck's formula for the black body radiation.

In case when the walls scatter the radiation homogenously in all directions i.e $(R_o \to \infty)$ $g_p(r,t)$ is given by

$$g_p(r,t) = X\int_{\upsilon_1}^{\upsilon_2}\alpha(\upsilon)G_p(t)I_{op\upsilon}(\upsilon)\ e^{\alpha(\upsilon)(r-R)}d\upsilon \tag{21}$$

Thus, generally $g_p(r,t)$ can be written as $g_p(r,t) = g_p(r)G_p(t)$

- Determination of the heat generation function of the laser beam to be amplified through the pumped rod g_L(r,z,t)

Considering the physical and optical parameter of the medium to be constant i.e. independent of the intensity of the laser radiation one gets:

$$I_{L\upsilon}(\upsilon, r, z, t)d\upsilon = I_{oL\upsilon}(\upsilon, r, 0, t)\exp(-((\alpha_L(\upsilon, r, t) - \beta_L(\upsilon, r, t))z)d\upsilon \tag{22}$$

where:

$I_{oL\upsilon}(\upsilon, r, 0, t)$ is the spectral intensity distribution of the laser beam incident in z direction on the pumped laser rod at z=0.

$I_{L\upsilon}(\upsilon, r, z, t)$ is the spectral intensity distribution of the beam at any location within the pumped rod.

α_L (υ ,r,t) and β_L (υ ,r,t) are the linear attenuation and amplification coefficient of the pumped rod in the frequency range of the incident laser radiation respectively.

The absorbed laser power density leading to heating the pumped rod is given from $\int_{\upsilon} \frac{dI_{L\upsilon}(\upsilon, r, z, t)}{dz} d\upsilon$ by

$$g_L(r, z, t) = \int_{\upsilon} I_{oL\upsilon}(\upsilon, r, 0, t)\alpha_h \exp(-(\alpha_L(\upsilon, r, t) - \beta_L(\upsilon, r, t))z)\, d\upsilon \qquad (23)$$

α_h is the coefficient responsible for the heating process and equal to 20 1/m[17].

with $I_{oL\upsilon}(\upsilon, r, 0, t) = \tilde{I}_o g_L(\upsilon, \upsilon_{oL}) g_L(r).G_L(t)$ and (24)

$$G_L(t) = \frac{(n+1)^{n+1}}{n^n} \frac{(t - \Delta t_o)}{\Delta t_L}(1 - (t - \Delta t_o)/\Delta t_L)^n \text{ , n = 3 , } 0 \le t \le \Delta t_L \qquad (25)$$

$$g_L(\upsilon, \upsilon_{0L}) = \frac{1}{\Delta\upsilon_{oL}} \sqrt{\frac{4\ln 2}{\pi}} \exp((\frac{\upsilon - \upsilon_{oL}}{\Delta\upsilon_{oL}})^2 4\ln 2) \qquad (26)$$

$$g_L(r) = e^{-2\frac{r^2}{w_o^2}} \quad \text{case of Gaussian distribution.} \qquad (27)$$

$$g_L(r, z) = \frac{w_0^{\,2}}{w(z)^2} e^{-2\frac{r^2}{w(z)^2}} \quad \text{case of Gaussian beam.} \qquad (28)$$

$$\text{with } w(z) = \sqrt{\frac{\lambda\tilde{L}}{2\pi n_{01}}} \sqrt{1 + \frac{(n_{02}\tilde{L} + 2(n_{02}d + n_{01}z))^2}{n_{02}^2\tilde{L}^2}} \qquad (29)$$

where: Δt_L is the pulse duration of the laser radiation, Δt_o is the time retardation leading to coinciding the maximum intensity of the laser radiation with the maximum of the pump beam, $\Delta\upsilon_{oL}$ is the full width at half maximum of the laser radiation, υ_{oL} is the center frequency of the laser radiation , w_o is the width of the laser beam , $\tilde{L}$ is the length of the resonator which is equal to the length of the rod inside the amplifier, λ is the laser wave length, n_{01} is the refractive index of the rod inside the resonator, n_{02} is the refractive index of the medium between the resonator and the amplifier, d is the distance between the laser resonator and the amplifier, w(z) is the width at any location z in the amplifier;

- Estimation of the temperature of the blackbody radiation

Considering the pumping process to be given by a blackbody radiation penetrating radialy the rod and initiating inversion population leading to a defined amplification at the maximum of the temporal distribution of the pump intensity, the central frequency of the spectral distribution of the laser radiation and the maximum of the spatial laser profile which is at r = 0.
For an amplification factor 10^n at z =L one gets from Eqn. (22)

$$\frac{I_L(\upsilon_{oL},r,L,\tilde{t})}{I_{o_L}(\upsilon_{o_L},r,0,\tilde{t})}=e^{-(\alpha_L(\upsilon_{o_L},r,L,\tilde{t})-\beta_L(\upsilon_{o_L},r,L,\tilde{t}))L}=10^n \qquad (30)$$

where υ_{o_L} is the central frequency of the laser radiation, $\tilde{t}$ is the time at which the pump radiation has its maximum value. With the spectral gain profile of the medium at $\upsilon=\upsilon_{oL}$ given by,

$$g_m(\upsilon,\upsilon_{oL})=\frac{1}{\Delta\upsilon_o}\sqrt{\frac{4ln2}{\pi}} \qquad (31)$$

$\beta_L(\upsilon_{o_L},0,L,\tilde{t})-\alpha_L(\upsilon_{o_L},0,L,\tilde{t})$ can be written as

$$\beta_L(\upsilon_{o_L},0,L,\tilde{t})-\alpha_L(\upsilon_{o_L},0,L,\tilde{t})=c^2\frac{A_{21}}{8\pi\upsilon_o^2}g_m(\upsilon,\upsilon_{oL})\left(N_2-\frac{g_2}{g_1}N_1\right)=\frac{1}{L}[n\ln 10] \qquad (32)$$

where g_2 and g_1 are the statistical weights, N_1 and N_2 are the densities of the atoms in the lower and upper level respectively.
Considering a three level system with g_2=g_1, one gets from the rate equations after setting the density of the laser radiation equal to zero at the threshold condition and N_2-N_1 from Eqn. (32),

$$N_o\frac{W_{13}-A_{21}}{W_{13}+A_{21}}=N_2-N_1=\frac{n\,ln10}{L}\sqrt{\frac{\pi}{4ln2}}\,\frac{8\pi\upsilon_o^2\,\Delta\upsilon_o}{c^2A_{21}} \qquad (33)$$

with $W_{13}=\rho(\upsilon)\,B_{13}$ one gets from the last equation

$$\rho(\upsilon)=\frac{A_{21}}{B_{13}}\,\frac{N_o+\frac{n\,ln10}{L}\sqrt{\frac{\pi}{4ln2}}\,\frac{8\pi h\upsilon_o^2\,\Delta\upsilon_o}{c^2A_{21}}}{N_o-\frac{nln10}{L}\sqrt{\frac{\pi}{4ln2}}\,\frac{8\pi h\upsilon_o^2\,\Delta\upsilon_o}{c^2A_{21}}} \qquad (34)$$

where A_{21} and B_{13} is the Einstein coefficient for spontaneous emission and absorption respectivly. N_o is the total number density of active atoms, $\rho(\upsilon)$ is the spectral density of the pump radiation.

Considering $\rho(\upsilon)$ being the spectral energy density of a black body manipulated through the optical system, which contains the pump source, the reflecting surface and the laser rod, $\rho(\upsilon)$ can be calculate if the variables of the RHS are known. Since all parameters of the RHS of Eqn. (34) are tabled except $\mathbf{B}_{13}$, it has to be estimated.

From the linear absorption coefficient at the maximum of the two absorption lines $^4F_1$ and $^4F_2$ one gets [17].

$$\frac{h\upsilon_{oI}}{c} B_{13I}\, N_o \frac{1}{\Delta\upsilon_I}\sqrt{\frac{4ln2}{\pi}} = 600\, m^{-1} \tag{35}$$

$$\frac{h\upsilon_{oII}}{c} B_{13II}\, N_o \frac{1}{\Delta\upsilon_{II}}\sqrt{\frac{4ln2}{\pi}} = 440 m^{-1} \tag{36}$$

where: υ_{oI} and υ_{oII} are the central frequencies of the absorption lines $^4F_1$ and $^4F_2$, respectively.

B_{13I} and B_{13II} are the Einstein absorption coefficients for the absorption lines.

$\Delta\upsilon_I$ and $\Delta\upsilon_{II}$ are the full width at half maximum of the absorption lines.

Knowing B_{13I} and B_{13II}, one get,

$$B_{13} = B_{13I} + B_{13II} \qquad \text{at } \upsilon = \frac{\upsilon_{oI} + \upsilon_{oII}}{2}$$

To determine the temperature of the black body which has to pump the laser rod to get at r = 0,z=L and $t = \tilde{t}$, the desired amplification 10^n, one has to set the black body spectral energy density distribution at $\upsilon = \frac{\upsilon_{oI} + \upsilon_{oII}}{2}$ after multiplication with $G_p(t = \tilde{t})$ and $g_p(r = 0)$ equals to Eqn. (34) to get:

$$T = \frac{h\upsilon}{k} \frac{1}{ln\left(\frac{N_o - \frac{n\,ln10}{L}\sqrt{\frac{\pi}{4ln2}}\frac{8\pi h\upsilon_o^2\,\Delta\upsilon_o}{c^2 A_{21}}}{N_o + \frac{nln10}{L}\sqrt{\frac{\pi}{4ln2}}\frac{8\pi h\upsilon_o^2\,\Delta\upsilon_o}{c^2 A_{21}}} \frac{B_{13}}{A_{21}} \frac{8\pi\, h\upsilon^3}{c^3} g_p(r=0) G_p(\tilde{t}) + 1 \right)} \tag{37}$$

Thus, $N_2 - N_1$ at the time and location at which the pump radiation is maximum, is given according to Eqn. (33) by

$$N_2 - N_1 = N_o \frac{B_{13}\rho(\upsilon)\,\tilde{g}_p(r=0) G_p(t=\tilde{t}) - A_{21}}{B_{13}\rho(\upsilon)\,\tilde{g}_p(r=0) G_p(t=\tilde{t}) + A_{21}} \tag{38}$$

Replacing $\tilde{g}_p(r=0)$ and $G_p(t=\tau)$ by $\tilde{g}_p(r)$ and $G_p(t)$, respectively, in Eqn. (38) one dets

$$\beta_L(\upsilon_{oL},r,L,t)-\alpha_L(\upsilon_{oL},r,L,t)=\sqrt{\frac{4ln2}{\pi}}\,\frac{1}{\Delta\upsilon_{oL}}\,\frac{c^2A_{21}}{8\pi\upsilon_{oL}^2}\,N_o\,\frac{\frac{B_{13}}{A_{21}}\rho(\upsilon)\tilde{g}_P(r)G_p(t)-1}{\frac{B_{13}}{A_{21}}\rho(\upsilon)\tilde{g}_P(r)G_p(t)+1}. \qquad (39)$$

3. Computation:

The physical and thermal parameters required in the calculation are taken from Table (1).

Table (1): Physical and thermal parameter of Ruby.

Symbol Unit	ρ $\frac{kg}{m^3}$	C_p $\frac{W.sec}{kg.^ok}$	N_o $\frac{ions}{m^3}$	υ_{01} Hz	υ_{21} Hz	υ_{22} Hz	υ_{31} Hz	υ_{32} Hz	$F_{\perp}^{21}$ 1/m
Value	4300	43	1.58E25	1.5E15	7.3E14	7.1E14	5.4E14	5.53E14	320
Symbol Unit	$F_{\perp}^{31}$ 1/m	$F_{\parallel}^{32}$ 1/m	$F_{\parallel}^{01}$ 1/m	$F_{\parallel}^{22}$ 1/m	$\Delta\upsilon_o$ Hz	k $\frac{W}{m^oK}$	H $\frac{W}{m^oK}$	p_{cr} W	n_o
Value	155	285	360	280	0.3298E12	42	10000	5000	1.759

$F_{\parallel\perp}^{ij}$ is the absorption coefficient parallel and perpendicular to the C –axis of the crystal

υ_{ij} are the corresponding central frequency to F^{ij}

N_o is the Cr^{3+} concentration

Calculation of the temperature distribution resulting from the pumping process $T_p(r,t)$:

To get $T_p(r,t)$ Eqn. (13) was calculated after substituting for $G_p(r,t\,|r',\tau),G_p(t)$ and $g_p(r',\tau)$ from Eqns. (12, 19 and 20), respectively. X in Eqn. (20) which represents the part of the absorbed radiation responsible for heating the rod was given the value 0.3 [17].

Calculation of the temperature distribution resulting from the laser radiation $T_L(r,z,t)$:

To get $T_L(r,z,t)$, Eqn. (16) was calculated after substituting for $G_L(z,t\,|z',\tau)$, $g_L(r',z',\tau)$ and $G_L(t)$ from Eqns. (15, 23 and 25), respectively. In equation (15) t is replaced by $(t-\tau)$.

The radial intensity distribution of the laser radiation was considered to be given by Eqn. (27) for the case of Gaussian distribution and Eqn. (28) for the case of Gaussian beam, where w(z) was taken from Eqn. (29).

The computation was carried out considering $w_o = 10^{-4}$m for the Gaussian distribution and w_o = 1.77 x 10^{-4}m for the Gaussian beam and a maximum monochromatic laser intensity $\tilde{I}_o$ given by $\tilde{I}_o = 10^{11} W/m^2$ at r = 0 and $t = \Delta t_p / 4$. The locations and the times at which the temperature was calculated are the multiple of the following Δz, Δr_i and Δt_i values:

Δz is considered to be given by $\Delta z = L/10$

Δr_i values are given as follows;

$\Delta r_1 = R_o / 10 \qquad 0 \le r \le R_o$

$\Delta r_2 = (w_o - R_o)/10 \qquad R_o \prec r \le w_o$ or $w_o \prec r \le R_o$

$\Delta r_3 = (R - w_o)/10 \qquad w_o \prec r \le R$

The time was sliced into four different intervals Δt_i ;

$\Delta t_1 =$ the time before initiating the laser pulse was divided by 5

$t = \Delta t_1 \times n_1 \qquad n_1 = 0......5$

$\Delta t_2 =$ the time during the laser pulse duration was divided by 16

$t = \Delta t_1 \times 5 + \Delta t_L / 16 \times n_2 \qquad n_2 = 1.......16$

$\Delta t_3 =$ the time calculated from the equation (the pump pulse duration – (the time before initiating the laser pulse + pulse duration) was divided by 5

$t = (\Delta t_p - (\Delta t_1 \times 5 + \Delta t_L))/5 \times n_3 + (\Delta t_1 \times 5 + \Delta t_L) \qquad n_3 = 1.....5$

The time after switching off the pumping process was given by

$\Delta t_4 =$ 2X pulse duration of the pump beam

$t = \Delta t_p + n_4 \times \Delta t_4 \qquad n_4 = 1......7$

Since the integration was carried out over z', r' and τ, Δz, Δr_i *and* Δt_i were subdivided into 6, 6 and 5 equal interval, respectively.

3. Results and Discussion:

- Heating Resulting from the Pump Beam:

Figure (1) represents the radial intensity distribution of the black body radiation (pump beam) within a ruby rod after integrating its spectral intensity distribution over the frequency with R_o as a parameter. The considered frequency interval covers the range of maximum absorption of the ruby rod

[17]. The calculation of the curves was carried out at $t = \Delta t_p/4$ according to the equation:

$$I(r,T) = \int_{\upsilon_1}^{\upsilon_2} I_{0bb}(\upsilon,T) e^{\alpha(\upsilon)(r-R)} \frac{R_0 + R}{R_0 + r} d\upsilon$$

where I_{obb} is the intensity distribution of the black body radiation as a function of r and T which leads to an amplification of 100 at r = 0, z = L and $t = \Delta t_p / 4$. The temperature fulfilling this condition have the values 5745, 8073, 9575 and 19421 K for $R_o = 10^{-4}$, 5×10^{-4}, 10^{-3} and ∞ m, respectively. This behaviour can be explained as follows. Since a particular population inversion along the z axis at r = 0 and in turn a particular intensity is needed to get the required amplification for each R_o value, therefore smaller temperature of the black body, results as the focusing becomes more pronounced i.e. R_o decreases. Due to the reduced focusing as R_o increases greater black body temperature is needed to initiate the intensity required for the population inversion along the z axis. This fact leads for $R_o = \infty$ m to greater intensity in the outer regions than in the axis of the rod.

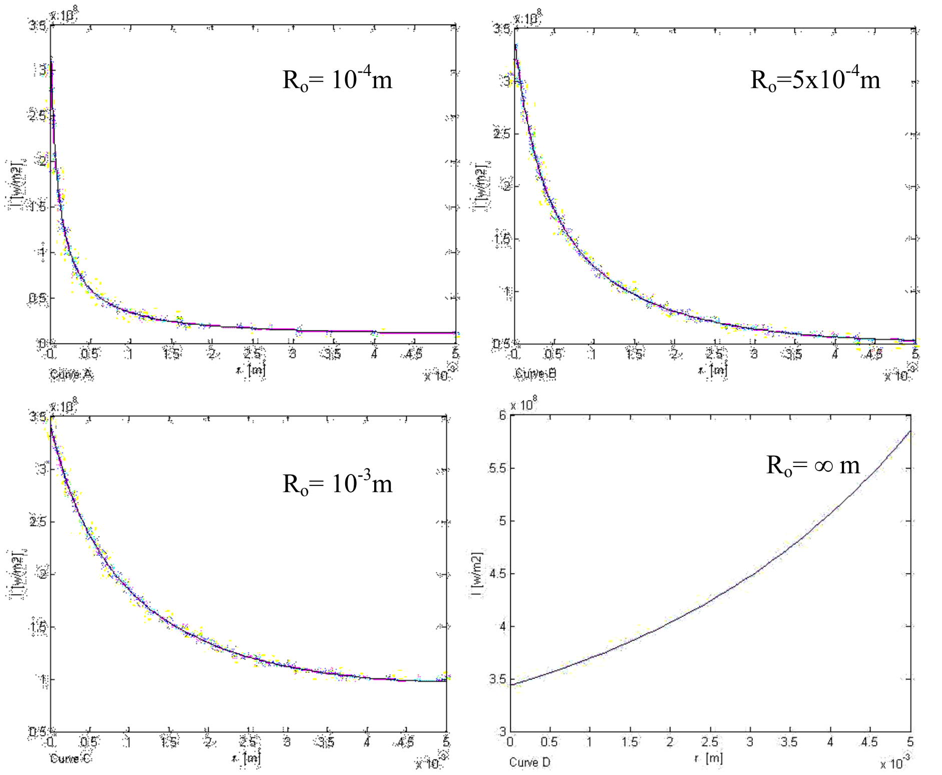

Figure (1): The radial intensity distribution of the black body radiation (pump beam) after integrating its spectral distribution over the frequency from $0.46X10^{15}$Hz to $1.5X10^{15}$Hz with R_o as a parameter. calculation was carried out at $t = \Delta t_p/4$.

From the Figure, it is also found that the gradient of the intensity in the vicinity of r = 0 is negative and smallest for $R_o = 10^{-4}$ m and greatest for $R_o = 10^{-3}$ m and that it changes its sign as R_o becomes infinite. This behavior can be attributed to the fact that since the radiation is incident in the radial direction towards the axis of the rod, it will suffer decay due to the absorption as r decreases. This decay will be over compensated as focusing is introduced.

Figure (2) represents the radial distribution of the heat generation function $g_p(r,t)$ calculated at $t = \Delta t_p/4$ with R_o as a parameter. The figure shows the same behavior as Fig. (1) except that their appears a relative minimum for all finite R_o values. It is more pronounced and more shifted towards smaller r values as R_o increases. This is because the function $\frac{R_0 + R}{R_0 + r} e^{\alpha(\upsilon)(r-R)}$,which is included in the radial intensity distribution of the pump beam as well as in $\frac{dI_{p\upsilon}(\upsilon)d\upsilon}{dr}$, has a minimum at $r = \frac{1}{\alpha(\upsilon)} - R_0$.

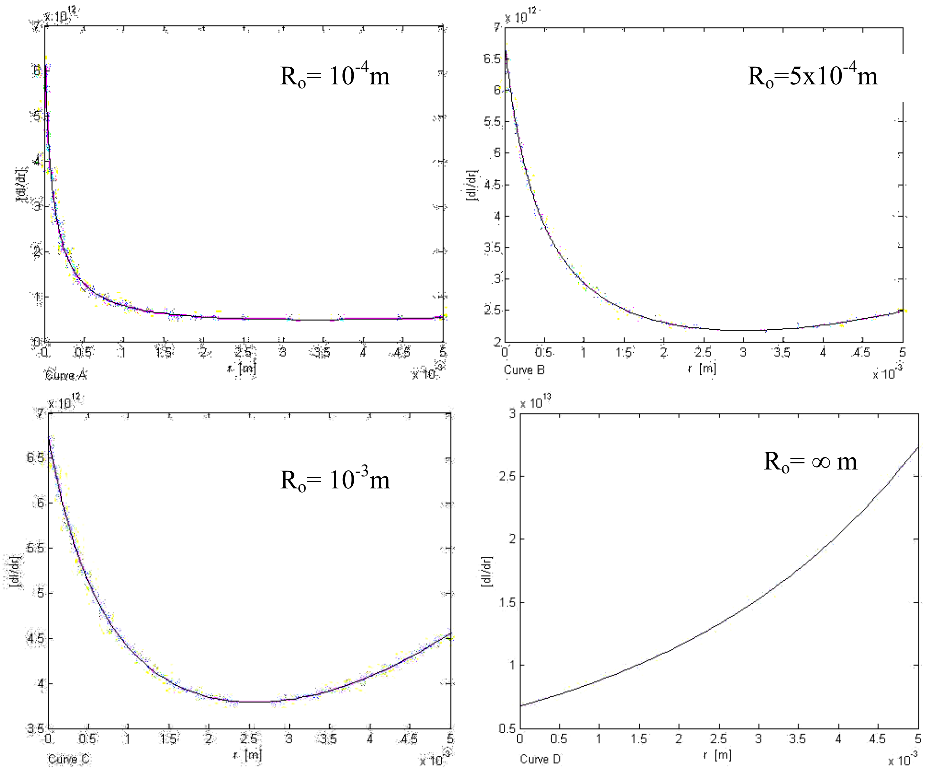

Figure (2): The heat generation function $g_p(r,t)$ resulting from the modified first derivative of the radial intensity distribution represented in figure (1) w.r.t. r with R_o as a parameter. The calculation was carried out at $t = \Delta t_p/4$.

The location of this minimum will, according to the above relation, be shifted towards smaller values of r as R_o increases. The increased pronunciation of the minimum for great R_o values might be due to mutual interaction between the focusing and attenuating effects. While the first effect $\frac{R_0 + R}{R_0 + r}$ leads to greater values of $g_p(r,t)$ at small values of r, leads the second one $e^{\alpha(\upsilon)(r-R)}$ to smaller $g_p(r,t)$ values at r = 0. Since as R_o increases the effect of focusing will decrease while that of the attenuation remains practically unchanged, thus the intensity at r=R will increase with increasing R_o and will attain its greatest value at $R_o = \infty$ m giving rise to an absolute minimum to appear at r = 0. The increase of $g_p(r,t)$ at r = R with increasing R_o values is due to the higher temperature of the pump beam which leads to shift the maximum frequency of the spectral intensity distribution towards greater frequencies where the absorption is also great. The constant value of $g_p(r,t)$ at r=0 is due to the considered constant amplification i.e. $(\beta_L - \alpha_L)$ at this location.

Figure (3) represents the time dependence of the temperature induced by the pump beam calculated at any z value, r= 0 with R_o as a parameter. From the figure it is evident that for all finite R_o values the temperature increases as the irradiation time of the pump beam increases. It reaches its maximum value at t = 0.6 x 10^{-3} sec which is greater than the time at which the intensity of the pump beam is maximum. This can be attributed to the fact that at the beginning of the pump pulse the temperature at the outer irradiated surface of the rod is small and, therefore, the cooling which is proportional to the temperature is negligible and has no effect on the temperature at r = 0. Moreover, the heat conductivity of the ruby and the gradient of the temperature everywhere along the radius are so small such that the absorbed radiation heats only the location where it is absorbed.

As the time of the irradiation increases the temperature increases and a greater gradient at r = 0 will build up. But because of the bad heat conductivity and the relatively small pulse duration the conducted energy remains small. This process lasts until the time at which the rate of the conducted heat energy into the cooler zones is equal to the rate of the energy of the absorbed radiation. At this time, the temperature reaches its maximum value after that the rate of the losses overcompensate the absorbed radiation and the temperature begins to decrease. Because of the great temperature gradient at the beginning of its reduction, the temperature decreases with a great slop followed by a smaller one till the end of the pump beam. After switching off the pulse of the pump beam the temperature reduces with a smaller rate. This behaviour is found to be valid for all finite R_o values.

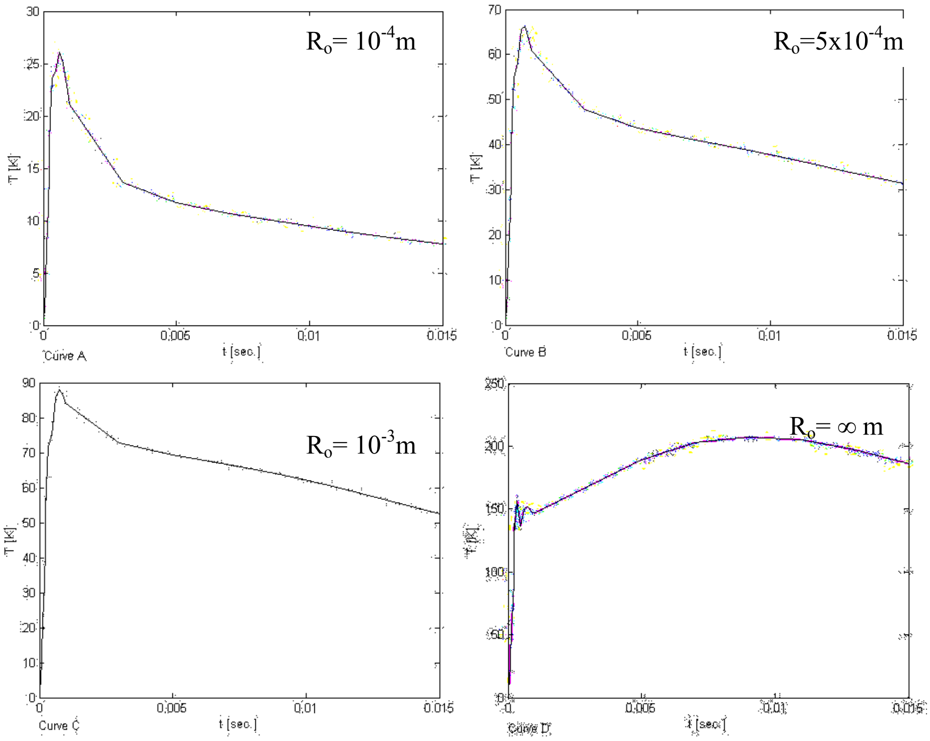

Figure (3): The temporal distribution of the temperature induced by the pump beam calculated at z = 0 , r = 0 with R_o as a parameter.

The curve representing $R_o=\infty$ m differs markedly from that of finite R_o values. The temperature is maximum at a much longer time after switching off the pump pulse. This behavior is due to the fact that greater amount of heat is stored in the outer regions where $\left|\frac{dI(r,T)}{dr}\right|$ is high. After switching off the pumping process the temperature tends to be homogenous across the cross-section of the rod leading to an increase of the temperature in the vicinity of r = 0. This process lasts until the heat conduction inside the material takes such small values, that the cooling of the outer regions overcompensates the conduction inside the material. At this time a monotone reduction of the temperature takes place.

Figure (4) illustrates the radial temperature distribution calculated at $t = \Delta t_p / 4$, arbitrary z value and R_o as a parameter. From the figure it is evident that for $R_o \neq \infty$ m the temperature due to the focusing effect is maximum at r= 0 and it decrease with decreasing R_o values. This ids due to the required constant $(\beta_L - \alpha_L)$ value at r= 0, which leads to a constant intensity in this location for each R_o value. Since the radial intensity distribution will be more flat in the vicinity of r (β_L - α_L)= 0 as R_o increases and since $\left|\frac{dT(r,t)}{dr}\right|$ is responsible for the heat conductivity to the cooler zones and this is, smallest for R_o= 10^{-3}m and greatest for R_o= 10^{-4} m, therefore the temperature in case of R_o =10^{-4}m is smaller than for R_o= 10^{-3}m. The increase of the temperature in the outer regions by increasing R_o values is due to the increase of $g_p(r,t)$ in that region by increasing these values.

The much higher temperature found in the outer regions of R_o= ∞ m is due to the greater value of $g_p(r,t)$ in these regions compared with that at r = 0. The higher temperature in the vicinity of r =0 than the cases of finite R_o values is due to the reason given for the curves having finite R_o values.

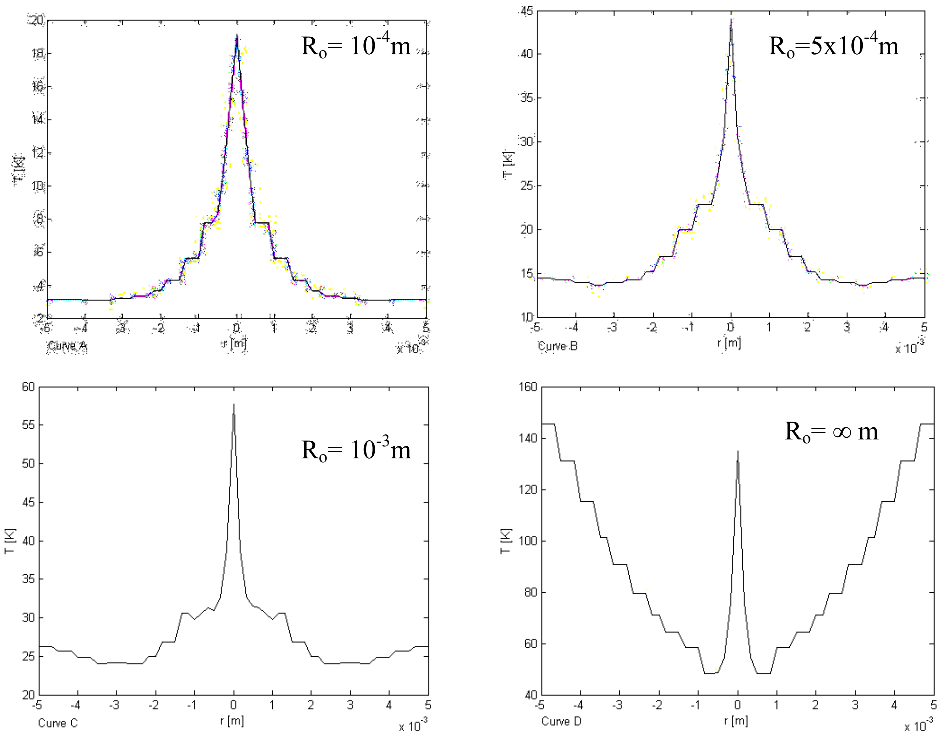

Figure (4): The radial temperature distribution induced by the pump beam calculated at t = $\Delta t_p/4$ at z = 0 with R_o as a parameter.

Figure (5) represents the radial distribution of (β_L - α_L) calculated at t =Δt_p /4 and R_o as a parameter. The observation of the curves shows that, for R_o= 10^{-4} m and $5x10^{-4}$ m (β_L - α_L) in the vicinity of r = 0 has a relative minimum shifted towards smaller r values as R_o increases, and that the minimum of the former R_o value is more pronounced than that of the latter one. The curves calculated for R_o= 10^{-3} m and ∞ m grow monotonically and exhibit specially for the case R_o=∞ m a saturated behavior at r = R. This behavior can be explained in view of the fact that the behavior $(\beta_L - \alpha_L)$ is dictated from the intensity distribution of the pump beam given by $\frac{R_0 + R}{R_0 + r} e^{\alpha(r-R)}$, which exhibits a minimum at $r = (1/\alpha)\ R_0$. Since the value of α is about 10^{-3} m^{-1}, it follows that by increasing R_o values the position of the minimum will shift towards smaller r values. As R_o is given values greater than or equal to 1/α the position of the minimum will shift to such small or negative r values where the relative minimum disappears. Since $(\beta_L - \alpha_L)$ at r = 0 must be 46.05 m^{-1}, this is because an amplification of 100 at z = L is set as a condition, and $(\beta_L - \alpha_L)$ depends on the pump energy density elsewhere, thus, a minimum laying to the right of r = 0 can have positive or negative values.

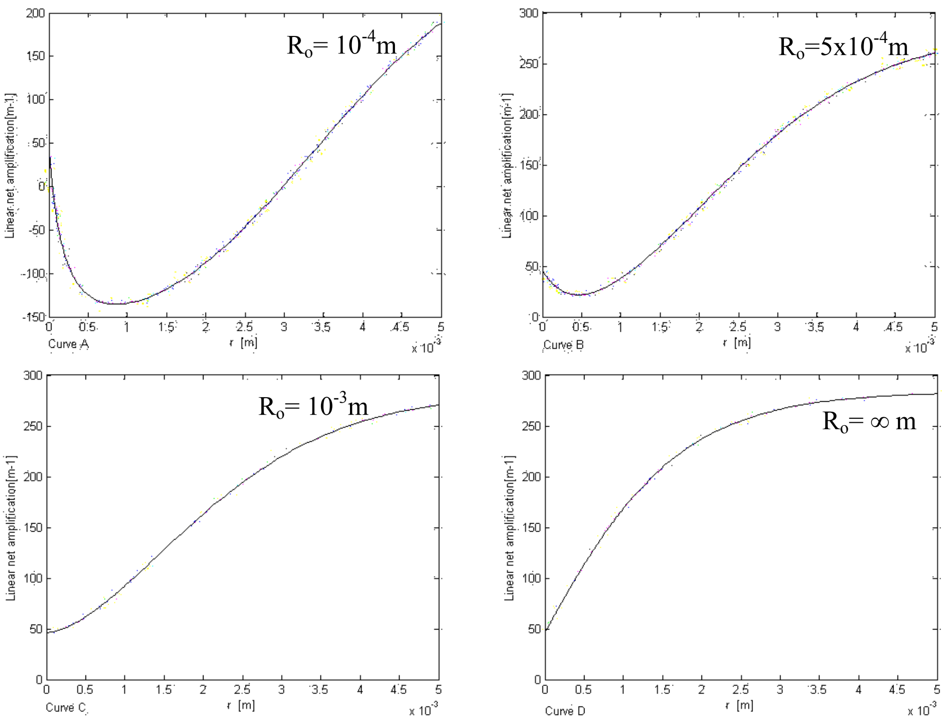

Figure (5): The radial distribution of the net amplification coefficient of the laser radiation (β_L-α_L) calculated at t = Δt_p/4 with R_o as a parameter.

In case of strong focusing the intensity of the pump radiation drops strongly with r leading to negative $(\beta_L - \alpha_L)$ values. The saturation at r=R for $R_0 = \infty$ m results from the behavior of $(\beta_L - \alpha_L)$ which is generally according to Eqn. (38) given by:

$$(\beta_l - \alpha_l) = c\,\frac{c_1 \dfrac{R_0 + R}{R_0 + r} e^{\alpha(r-R)} - 1}{c_1 \dfrac{R_0 + R}{R_0 + r} e^{\alpha(r-R)} + 1} \quad .$$

- Heating resulting from the laser beam of intensity $\tilde{I}_o = 10^{11} w / m^2$ **:**

- Case of Gaussian distribution of width W = 10^{-4}m

Figure (6) represents the temporal temperature distribution calculated at z=0, r =0 and R_o as a parameter. The figure shows that the maximum of the temperature occurs at a time greater than that at which the laser radiation is maximum i.e. $t = \Delta t_p / 4$. This is because at the beginning of the laser radiation more energy will be stored than transferred to the surrounding. This effect leads to an increase of the temperature. At times at which the radiation becomes smaller, such that the converted radiation into heat equals the conducted one, the temperature attains maximum value. As the time goes on and the radiation becomes smaller, the heat conduction becomes greater than the converted laser radiation into heat and the temperature reduces monotonically.

That the temperature is independent of R_o and depends only on w_o, as seen from the calculation, is an indicator that the temperature results from the absorbed laser radiation in the neighborhood of $z = 0$ and $r = 0$ although it is calculated through integration of the function $e^{(\beta_L - \alpha_L) z'} e^{-2\frac{r'^2}{w_o^2}}$ over all r' and z' values ranging respectively from $r' = 0$ to $r' = R$ and $z' = 0$ to $z' = L$. Since the exponent of the exponential function including z' varies with the time so due to this fact its effect will be more reduced than $e^{-2\frac{r'^2}{w_o^2}}$ which is time independent and depends only on w_o.

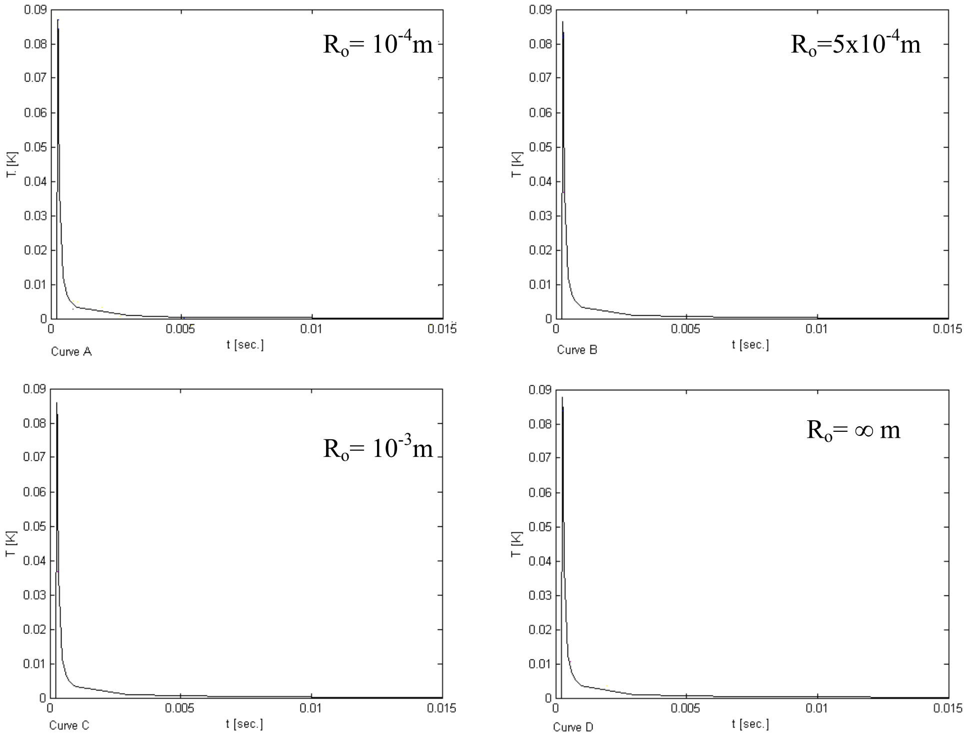

Figure (6): The temporal temperature distribution induced by a laser beam having a Gaussian spatial intensity distribution of width $w_o = 10^{-4}$m and pulse duration $\Delta t_L = 10^{-4}$ sec calculated considering linear behavior at z = 0 , r = 0 and R_o as a parameter.

Figure (7) represents the temperature versus time calculated at Z=L, r = 0 and different R_o values as a parameter. Also here the maximum of the temperature, due to the above cited reason, occurs at higher t values than $\Delta t_p / 4$. Due to the radial distribution of $(\beta_L - \alpha_L)$ given in figure (5) which leads to a greater radial amplification of the laser radiation as R_o increases, the temperature increases with increasing R_o values. Due to the small w_o value which will not be effectively reduced from the negative value of $(\beta_L - \alpha_L)$ for $R_o = 10^{-4}$m and the amplification of the laser radiation, the temperatures for all Ro values are greater than the corresponding ones of Fig. (6).

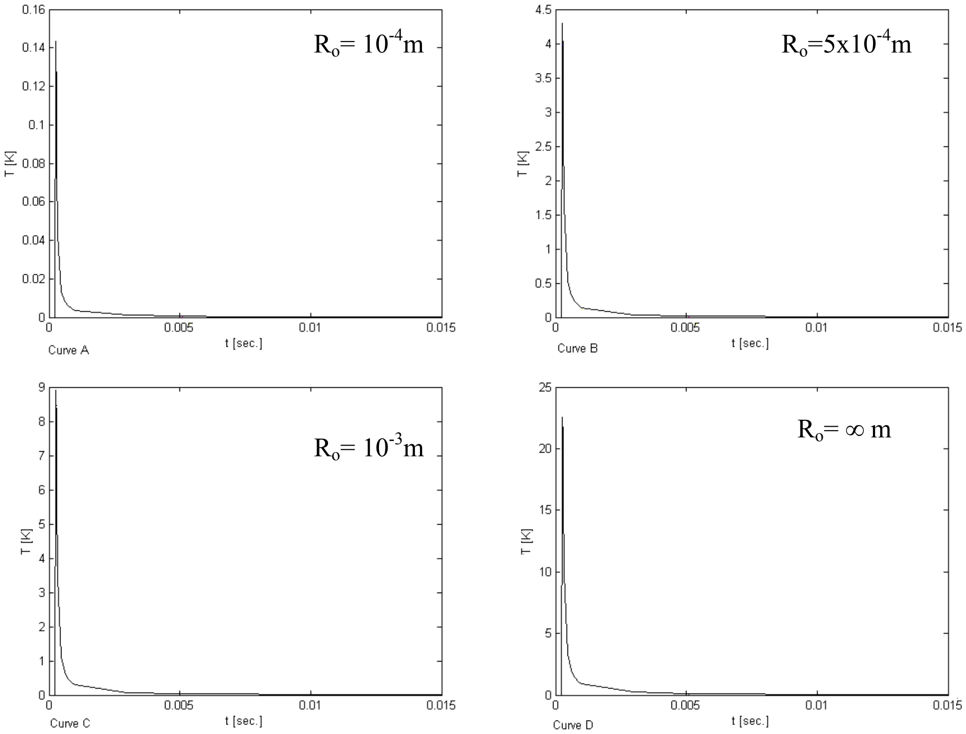

Figure (7): The temporal temperature distribution induced by a laser beam having a Gaussian spatial intensity distribution of width $w_o = 10^{-4}$m and pulse duration $\Delta t_L = 10^{-4}$ sec calculated considering linear behavior at z = L , r = 0 and R_o as a parameter.

Figure (8) represents the temperature distribution along the z axis calculated at $t = \Delta t_p / 4$, r = 0 and R_o as a parameter. From the figure it is evident that the temperature for $R_o = 10^{-4}$ m exhibits a minimum at z= 0.03 m and that for $R_o > 10^{-4}$m it increases monotonically with increasing z values. Moreover it shows that the temperature at z = L increases with increasing R_o values. The first behavior is due to the fact that for $R_o = 10^{-4}$ m the radial distribution of $(\beta_L - \alpha_L)$ exhibits negative values in the wings of the spatial distribution of the laser radiation. This leads in the vicinity of z = 0 to smaller amplification for $r<10^{-4}$m and reduction in the radial distribution of the laser radiation due to the negative $(\beta_L - \alpha_L)$ for $r > 10^{-4}$ m as well as a conduction of heat in the cooler parts adjusting to r = 0. Due to this fact the temperature will firstly decrease with increasing z values.

By further increase of z the wings become strongly reduced and will practically play no role in the heating process. The resulting laser radiation with the markedly smaller half width will, due to $(\beta_L - \alpha_L)$ radial distribution, be amplified and over compensates the heat conduction in the cooler parts leading to an increase of the temperature. That for all other R_o values, the temperature increases monotonically with increasing z values is due to the positive radial distribution of $(\beta_L - \alpha_L)$ which heats the surrounding of r = 0 more or less than that at r = 0. The increased value of temperature with increasing R_o values is due to the increased value of $(\beta_L - \alpha_L)$ in the wings of the laser radiation as R_o increases.

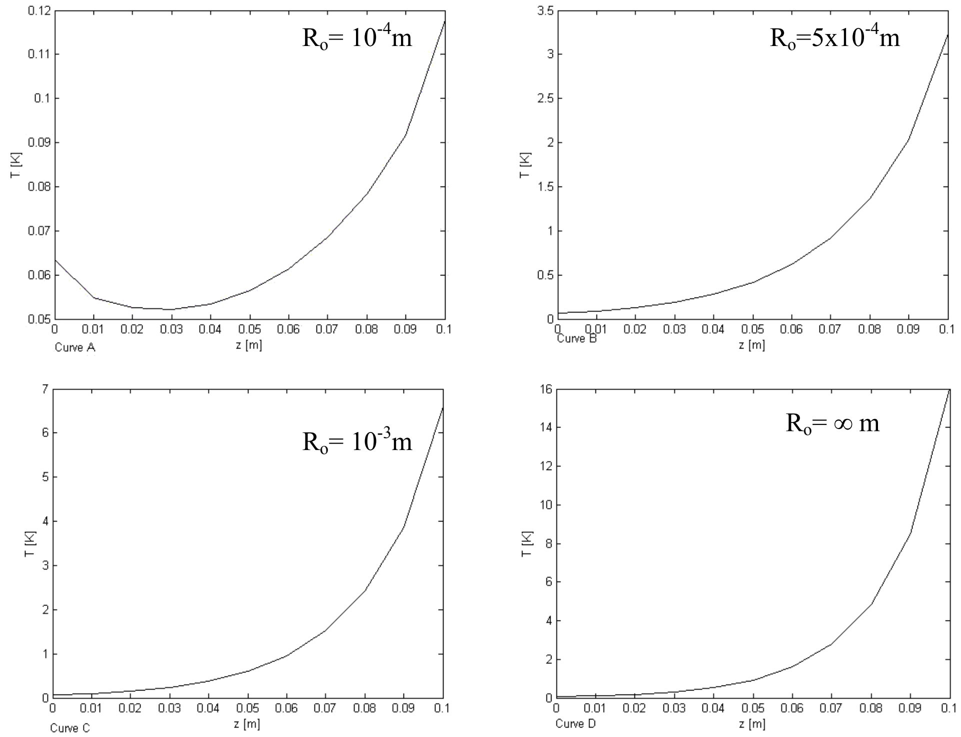

Figure (8): The temperature distribution as a function of z induced by laser beam having a Gaussian spatial intensity distribution of width $w_o = 10^{-4}$m and pulse duration $\Delta t_L = 10^{-4}$ sec calculated considering linear behavior at $t = \Delta t_p/4$, r = 0 with R_o as a parameter.

Figure (9) represents the radial temperature distribution calculated at $t = \Delta t_p / 4$, z= L with R_o as a parameter. Due to the reasons given in figure (8) belonging to the radial distribution of $(\beta_L - \alpha_L)$ for different R_o values the temperature at r = 0 increases with increasing R_o values. That the radial distribution of the temperature is broader than that of the incident radiation is for $R_o=10^{-4}$m due to the heat conduction. For $R_o=5x10^{-4}$m and 10^{-3}m it is due to the heat conduction and the radial distribution of $(\beta_L - \alpha_L)$. Because of the great positive slope of $(\beta_L - \alpha_L)$ in the case of homogenous illumination the radial half width in this case is broader than the others. The appearance of the step in the radial distribution comes from the variation of the r – values at which the temperature was calculated. The difference between two successive r values Δr was small in the vicinity of r = 0. As r was increased Δr was increased twice.

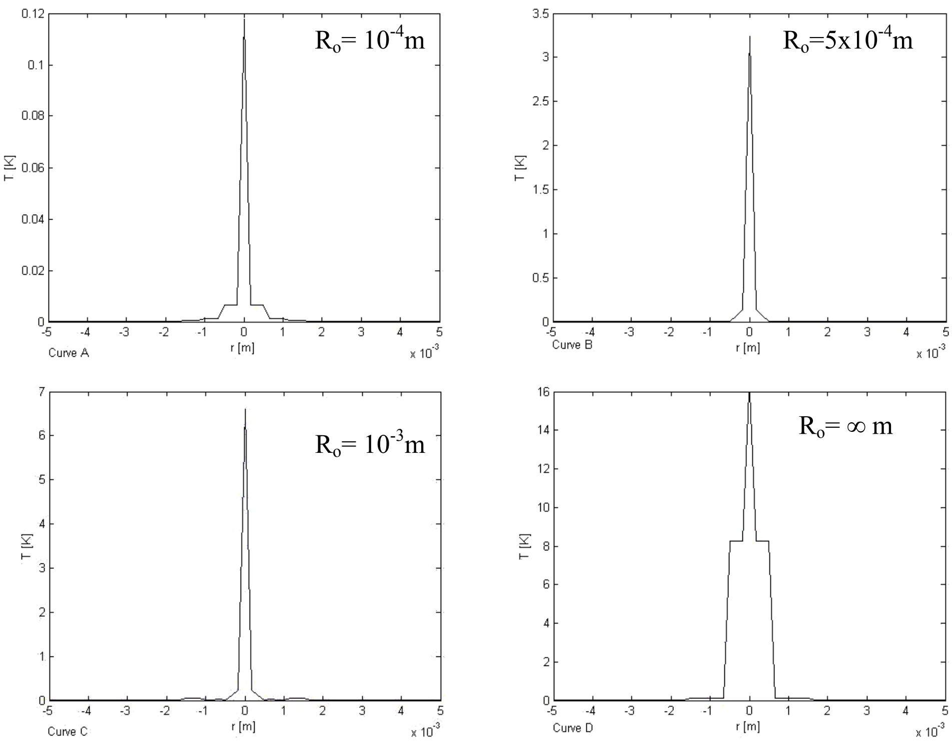

Figure (9): The radial temperature distribution induced by laser beam having a Gaussian spatial intensity distribution of width $w_o = 10^{-4}$m and pulse duration $\Delta t_L = 10^{-4}$ sec calculated considering linear behavior at $t = \Delta t_p/4$, z = L= 0.1m with R_o as a parameter.

- Case of Gaussian Beam:

Figure (10) represents the variation of the w(z) values along the z axis. It shows that w(z) at z = 0 is equal to 1.77 x 10^{-4}m and w(z) at z=L is equal to 3.9X10^{-4} m. The calculation of the temporal temperature distribution carried out at z = 0, r = 0 and different R_o values as a parameter, shows that the results, as seen and explained in Fig. (6), are independent of R_o. They have the same behavior as the curves of Fig. (6) but with about 3.75 times higher temperature. This is due to the greater area of the incident laser radiation which caries about 3.2 times higher power.

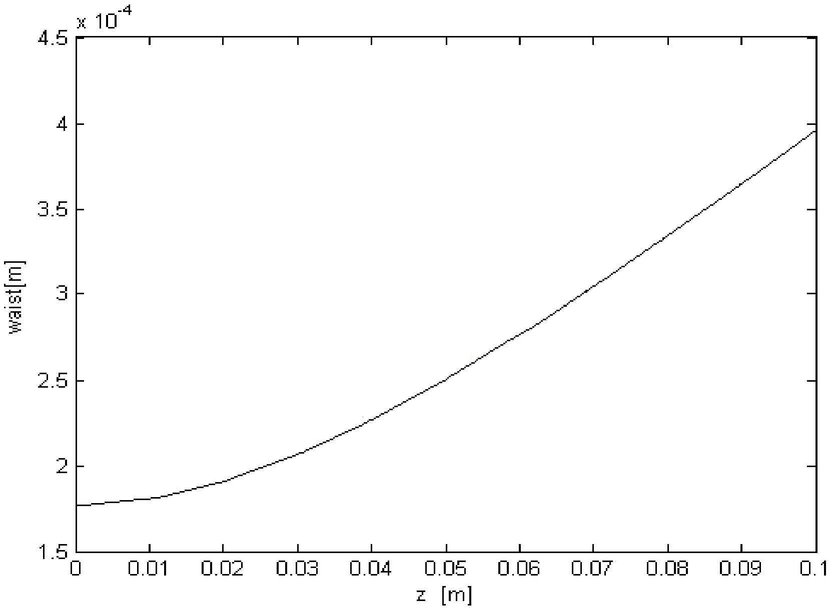

Figure (10): The width w as a function of z of a laser beam having a Gaussian intensity distribution considering linear case.

Figure (11) represents the temperature versus time calculated at r = 0, z = L with R_o as a parameter. That the temperature at z=L for R_o= 10^{-4}m is smaller than the temperature at z = 0, is due to the much broader spatial laser radiation at z = 0 than at z = L which results, as seen from Fig. (5), from the negative $(\beta_L - \alpha_L)$. Although the laser radiation at r = 0 will be amplified by factor 100 it seems that the reduction of the intensity due to the increasing width of the laser radiation and the truncation of the wings which occupy a great area is more effective than the small area which is amplified. Due to the radial distribution of $(\beta_L - \alpha_L)$ which leads to a greater radial amplification of the laser radiation as R_o increases, the temperature increases with increasing R_o values.

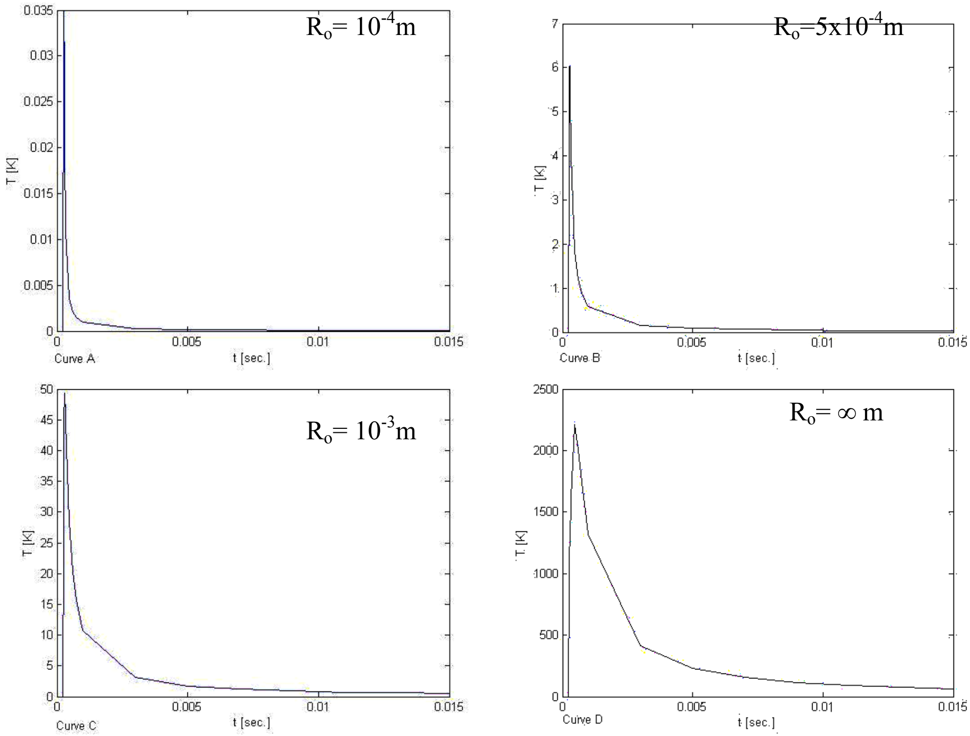

Fig. (11): The temporal temperature distribution induced by a laser beam having spatial intensity distribution of increasing width with increasing z – value (Gaussian beam) originating from a laser oscillator 0.1 m long, laying 0.1m a part from a laser amplifier. The Gaussian beam has a width of $1.77X10^{-4}$m at the input of the laser amplifier. The calculation was carried out considering linear behaviour at z = L, r = 0 and R_o as a parameter.

Figure (12) represents the temperature distribution along the z axis calculated at $t = \Delta t_p / 4$, r = 0 and R_o as a parameter. The figure shows for all R_o values except for R_o= 10^{-4}m the same behavior as seen in the previous case except that the temperature in this case, due to the increased w value with increasing z and the behavior of $(\beta_L - \alpha_L)$ which allows amplification that overcompensates the reduction of the intensity, is greater than the corresponding curves of w = 10^{-4} m. In contrary to the previous case the temperature calculated for R_o= 10^{-4} m decreases monotonically with increasing z values. This behavior is due to the fact that by increasing the z values the cross section of the laser radiation grows leading to reducing its intensity and to covering greater and greater zones of the negative $(\beta_L - \alpha_L)$ values. These effects lead to the decrease in the intensity and therefore in the temperature values.

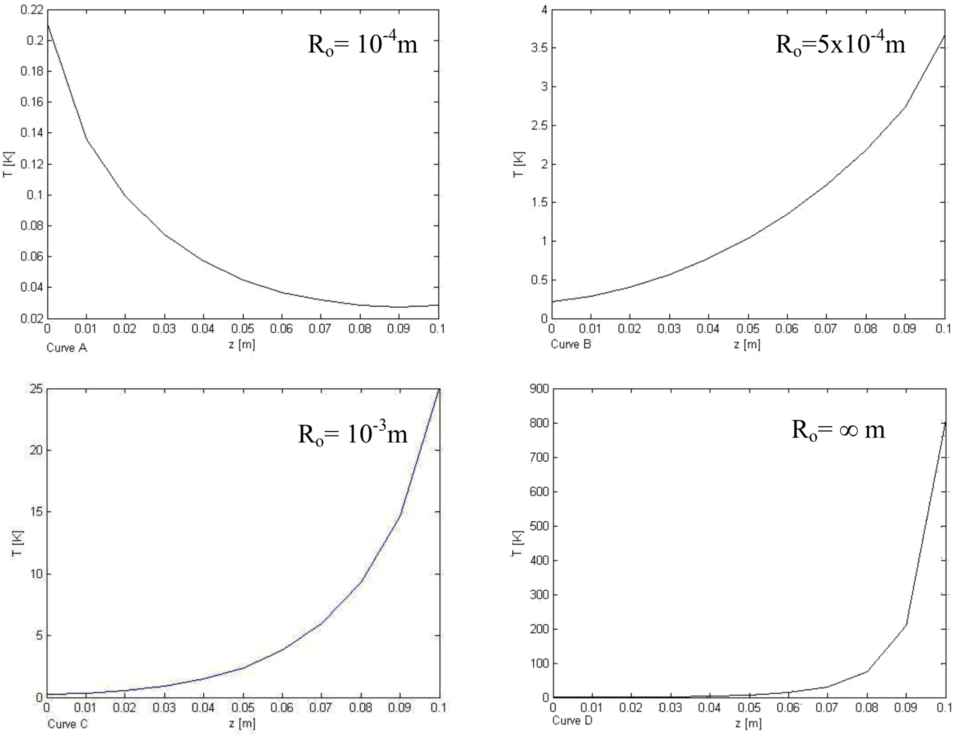

Figure (12): The temperature distribution as a function of z induced by a laser beam having spatial intensity distribution of increasing width with increasing z – value (Gaussian beam) originating from a laser oscillator 0.1 m long, laying 0.1m a part from a laser amplifier. The Gaussian beam has a width of $1.77X10^{-4}$m at the input of the laser amplifier. The calculation was carried out considering linear behavior at $t = \Delta t_p/4$, r = 0 and R_o as a parameter.

Figure (13) gives the radial distribution of the temperature calculated at $t = \Delta t_p / 4$, z = L and R_o as a parameter. The curves in the figure behave as the previously calculated ones. They differ only in the absolute value of the temperature and the appearance of a dip around r = 0 at $R_o = \infty m$. This behavior can be explained as follows: at r = 0 the amplification of the great laser intensity and the accumulation of heat generation at small times leads to the appearance of a peak at this location. As r increases the decay of the intensity of the laser radiation cannot be compensated from the increased value of $(\beta_L - \alpha_L)$. This behavior lasts till $(\beta_L - \alpha_L)$ takes such values able to compensate the reduction of the intensity where a minimum appears. By further increase of r, $(\beta_L - \alpha_L)$ will take values great enough leading to over compensate the decrease of the radial radiation. This behavior is continued till the appearance of a maximum after which the temperature begins to decrease.

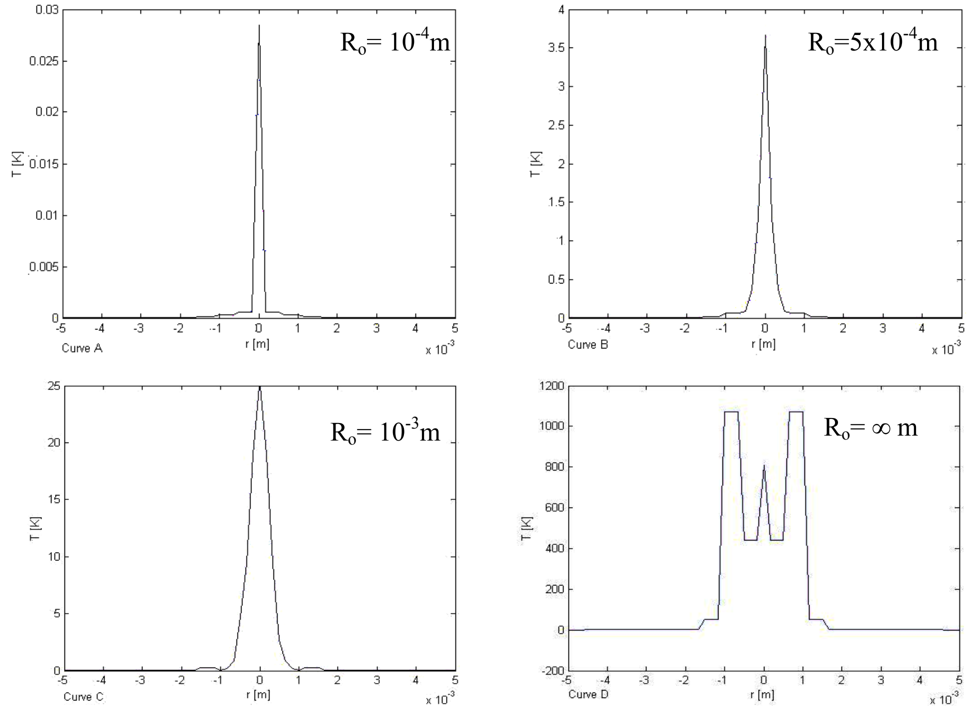

Fig. (13): The radial temperature distribution induced by a laser beam having spatial intensity distribution of increasing width with increasing z – value (Gaussian beam) originating from a laser oscillator 0.1 m long, laying 0.1m a part from a laser amplifier. The Gaussian beam has a width of $1.77X10^{-4}$m at the input of the laser amplifier. The calculation was carried out considering linear behavior at $t = \Delta t_p/4$, $z = L$ and R_o as a parameter.

4. Conclusions:

The focusing of the pump beam characterized by R_o values leads to:

- Effects concerning the pump source itself such as the dissipated energy, cooling and life time: Small R_o values lead to small temperature of the pump source and, thus, small energy consumption, reduced cooling problems and great life time.
- Effects concerning the interaction of the pump beam with the amplifying medium such as temperature and $(\beta_L - \alpha_L)$ radial distributions: Small R_o value leads to concentration of the temperature and to positive values of the

amplification factor only around r = 0. Great R_o values increase the value of the radial distribution of $(\beta_L - \alpha_L)$ and the temperature at greater r values.

The drastic decrease of the radial distribution of $(\beta_L - \alpha_L)$ for R_o = 10^{-4}m, which after reaching the required value of about 50 m^{-1} at r = 0, takes at r ≈ 10^{-4} m negative vales leads to decrease the width of the laser radiation during its propagation along the rod. As R_o was increased the wings of the laser radiation were more amplified than the parts located around r = 0. The calculated temporal temperature distributions at r = 0, z = 0 with R_o as a parameter are found to be practically independent of R_o and depend only on the radial width of the laser radiation and its shape. An increase of the temperature with increasing z for all R_o and w values except R_o = 10^{-4} m. For R_o = 10^{-4} m and $w \neq 10^{-4}$ m the temperature decreases with increasing z values. For R_o =10^{-4} m and w=10^{-4}m the temperature attains a minimum. The radial distribution of the temperature depends on R_o and w and attains for great w and R_o values beside higher temperature the appearance of side maxima.

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